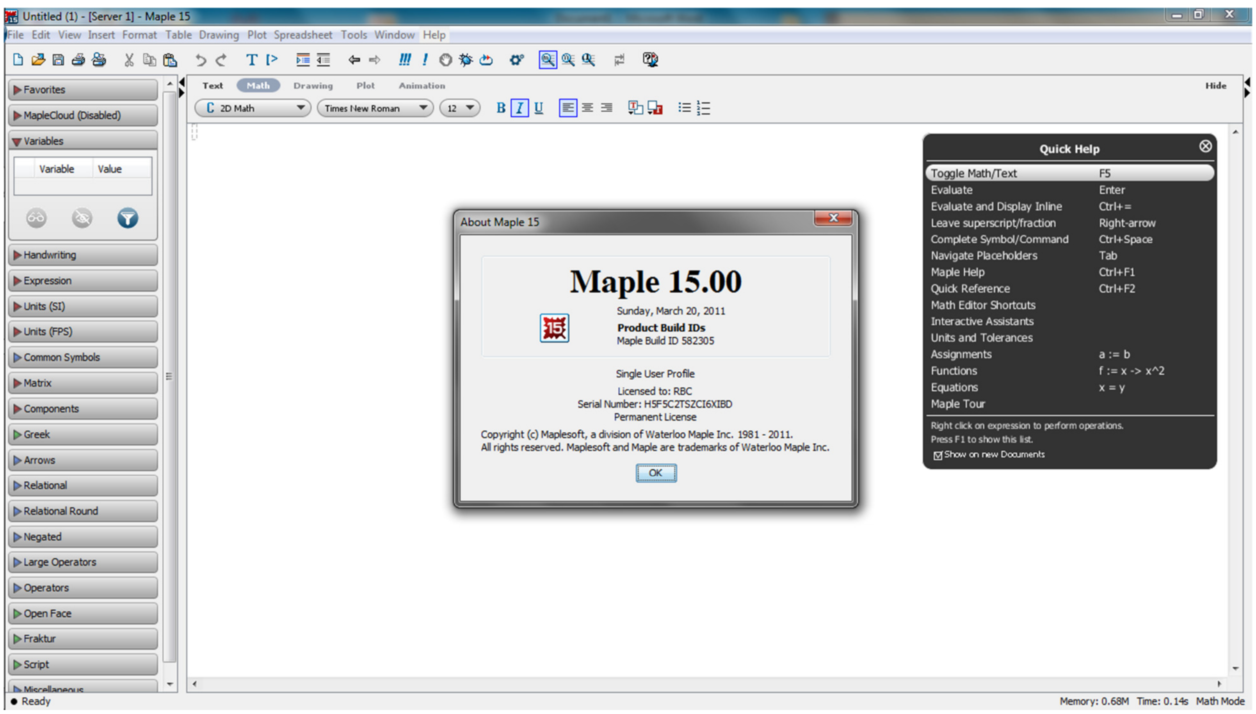
Chapter 5. Business Solutions using MAPLE

1. About Maple

It was first developed in 1980 by the Symbolic Computation Group at the University of Waterloo in Waterloo, Ontario, Canada. Since 1988, it has been developed and sold commercially by Waterloo Maple Inc. (also known as Maplesoft), a Canadian company also based in Waterloo, Ontario. The current major version is version 16 which was released in March 2012. DePaul University has a site license for computer labs, so you are free to use the program at any computer labs. Current available version at DePaul University is Maple 15 for Windows.

To run Maple, click START -> All Programs -> Mathematics -> Maple 15 (This can be different depending upon the configuration). Here is quick view of Maple 15.



2. Important Maple Statements

2.1 Maple Statements

There are many types of valid statements. Examples include statements that request help on a particular topic, display a text string, perform an arithmetic operation, use a Maple library routine, or define a procedure. Most Maple statements must have a trailing semicolon (;) or colon (:). If you enter a statement with a trailing semicolon, for most statements, the result is displayed. However, if you enter a statement with a trailing colon, the result is computed but not displayed.

```
> 2 + 3;
5
> 2 + 3:
```

2.2 Getting Help

To view an online help page for a particular topic, enter a question mark (?) followed by the corresponding topic name. For example, ?procedure displays a help page that describes how to write a Maple procedure.4 This type of Maple statement does not have a trailing colon or semicolon.

2.3 Displaying a Text String

The following statement returns a string. The text that forms the string is enclosed in double quotes, and the result (the text string) is displayed because the statement has a trailing semicolon. In the second example, no result is displayed because the statement has a trailing colon.

```
> "GSB 420 Class Calculus Review";
```

"GSB 420 Class Calculus Review"

2.4 Performing an Arithmetic Operation

The arithmetic operators in Maple are + (addition), - (subtraction), * (multiplication), / (division), and ^ (exponentiation). A statement can be an arithmetic operation that contains any combination of these operators. The standard rules of precedence apply.

```
> (100*20)/(23445-56);
```

$$\frac{2000}{23389}$$

Maple displays the result—in this case an exact rational number—in the worksheet or on the terminal in use, displaying the result as closely to standard mathematical notation as possible. To find the numbers with decimal points:

```
> evalf(100*20)/(23445-56);
```

0.08551028261

2.5 Assigning to a Name

By naming a calculated result or complicated expression, you can reference it. To assign to a name, use the assignment operator, :=

> $a := 500/175;$

$$\frac{20}{7}$$

> $y := 5 \cdot x + 10$

$$5x + 10$$

2.6 eval

eval is an useful command to find values for a defined equation as follows:

> $Eq := X^2 + 2 \cdot X + 10;$

$$X^2 + 2X + 10$$

> $eval(Eq, X = Y);$

$$Y^2 + 2Y + 10$$

> $eval(Eq, X = 10);$

$$130$$

2.7 solve

solve command can be used to find system equation solutions.

> $solve(\{2 \cdot x - y = 2, x + 2 \cdot y = 1\}, \{x, y\});$

$$\{x = 1, y = 0\}$$

> $solve(\{10 \cdot x - 2 \cdot y = 2, 2 \cdot x + 5 \cdot y = 1\}, \{x, y\});$

$$\left\{x = \frac{2}{9}, y = \frac{1}{9}\right\}$$

2.8 Unassigning a Name

There are many ways to unassign a name (reset its value to its name).

If a name is unassigned, it acts as an unknown. After a name is assigned a value, it acts as a variable. However, it is often desirable to unassign an assigned name, so that you can use the name as an unknown again.

One method of unassigning a name is to assign the unevaluated name (its initial value) to the name. To do this, enclose the name in right single quotes to delay evaluation.

```
> a := 4;
a := 4
> a;
4

> a := 'a';
a := a

> a;
a
```

You can also unassign names by using the unassign command.
unassign(name1, name2, ...);

> unassign('i');

The value returned by unassign is NULL.
Another way you can unassign a name is by using the evaln command.
evaln(name);

The evaln command evaluates name to a name (as opposed to eval- uating name to its value as in other calls). Therefore, you can unassign a name by using evaln in the following manner.
> a := 100 ;
a := 100

> a := evaln(a);
a := a

> a;
a

2.8 Clearing the Maple Internal Memory

Clear the internal memory during a Maple session by entering the **restart** command or clicking the restart icon[18] on the toolbar of the worksheet (in GUI versions). When you enter this command, the Maple session returns to its startup state; all identifiers (including variables and procedures) are reset to their initial values.[19]

> restart:

3. Calculus using Maple

One of the uses of calculus is to determine the extreme points of a curve. While a graph can be used to estimate the highest and lowest points, calculus can be used to determine the values to any desired degree of accuracy.

3.1 diff function

The structure of the command is diff(equation, variable). Suppose we want to find a derivative for following function:

>$y := 1.5\,x^3 - 20\,x^2 + 5\cdot x + 500$

The diff statement will give the derivatives as like:

> $diff(y, x)$

$$4.5x^2 - 40x + 5$$

If you want to find second derivative using diff, but x being substituted by x$2, which means you are doing the second derivative:

> $dyy := diff(\mathrm{y}, \mathrm{x\$2});$

$$9.0x - 40$$

3.2 Partial Derivative Example

$$dyyy$$

$y := 25x_1^2 + 5x_1 \cdot x_2 + 7x_2^2;$

$$25x_1^2 + 5x_1x_2 + 7x_2^2$$

$diff(y, x_1);$

$$50x_1 + 5x_2$$

$diff(y, x_2);$

$$5x_1 + 14x_2$$

To make a variable with subscript number, use "_" before the subscript. For example, x_1 will make x_1 .

3.2 Optimization using Maple

One of the uses of calculus is to determine the extreme points of a curve. While a graph can be used to estimate the highest and lowest points, calculus can be used to determine the values to any desired degree of accuracy. By solving the first derivative, we can identify the switching points that can be potentially maximum or minimum value. From the above example, we have

> $solve(dy, x);$

$$8.762079828, 0.1268090605$$

The statement shows that the first derivative has a solution of 0.1268090605 and 8.762079828. To confirm the maximum or minimum, we can exam the second derivative as

> $eval(dyy, x = 8.762079828);$

$$38.85871845$$

> $eval(dyy, x = 0.1268090605);$

$$-38.85871846$$

For x= 0.1268090605, dyy = -38.85… (means local maximum) and x=8.76… dyy = 38.85… (means local minimum)

3.3 Lagrange Multipliers

1. Your company sells DVDs in two regions: 1 and 2. Your sales staff came up with the following relationships between sales and advertising expenses in each of these regions:

$$S_1 = 50 + 10A_1 - A_1^2$$
$$S_2 = 20 + 5A_2 - 0.5A_2^2$$

where S_i = sales revenue in Region "i" (millions of \$),

A_i = advertising expenditures in Region "i" (millions of \$).

Your boss (or you) provides a total advertising budget of \$7 million. Identify the advertising expenses for each region that will maximize your company's total sales (from these two regions).

<u>Solution using MAPLE:</u>

Doing Lagrange by hand is a long process, but with Maple it can be done with one command. But we first need to activate the Multivariate Calculus package:

> $with(Student[MultivariateCalculus]):$

> $Sales := 50 + 10 \cdot A1 - A1^2 + 20 + 5 \cdot A2 - 0.5\,A2^2;$

$$70 + 10\,A1 - A1^2 + 5\,A2 - 0.5\,A2^2$$

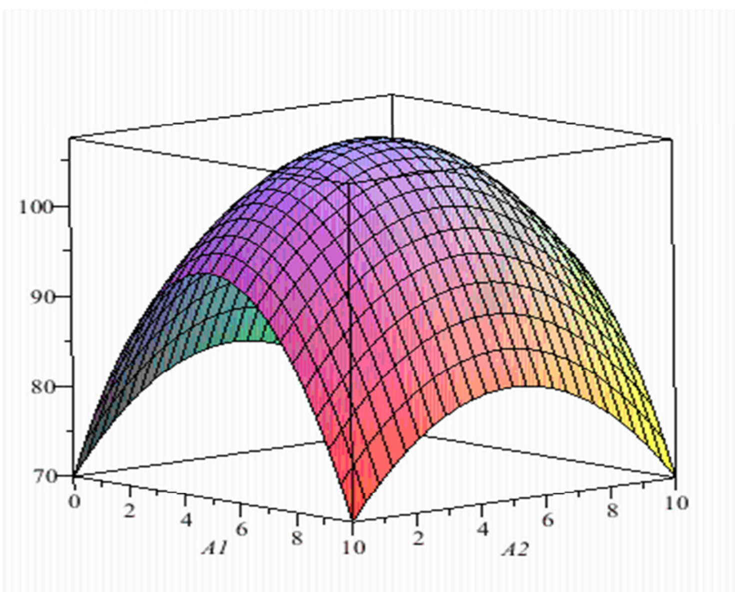
> $LagrangeMultipliers(Sales, [7 - A1 - A2], [A1, A2]);$

$$[4., 3.]$$

> $eval(Sales, [A1 = 4, A2 = 3]);$

$$104.5$$

$plot3d(Sales, A1 = 0..10, A2 = 0..10);$



The results show the advertisement expenditure in 1 area should be 4 million dollars and 3 million dollars in Area 2, which makes the maximum sales of 104.5 million dollars.

4. Generating Graphs

As we see from previous example, Maple graphs help to understand the overall shape of function before or after the mathematical solutions. Here are some graphic functions to be used in business world.

4.1. plot

A graph helps to understand the overall structure of a function. The equation can be included in the plot command or the predefined equation can be called in plot command.

1) Example of simple graph

> $plot\left(1.5 \cdot x^3 - 20 \cdot x^2 + 5 \cdot x + 500\right)$;

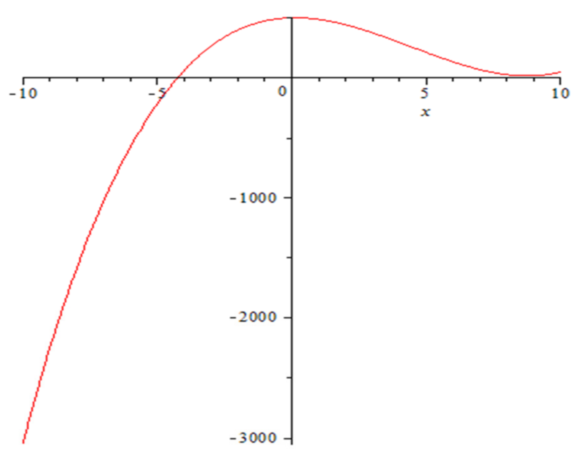
or

> $y := 1.5\,x^3 - 20\,x^2 + 5 \cdot x + 500$

$$1.5\,x^3 - 20\,x^2 + 5\,x + 500$$

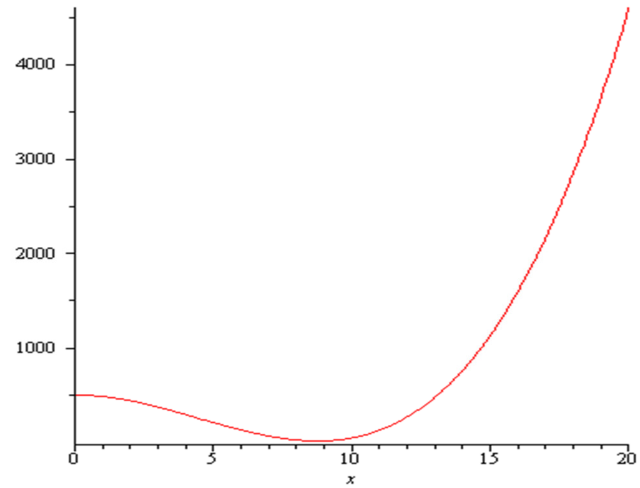
> $plot(y)$;

Both will generate the same graph as following graph:



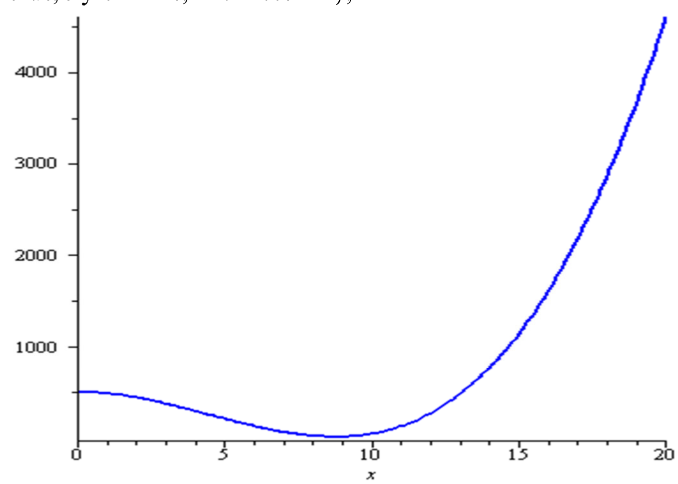
2) Graph with X range

> $plot(y, x = 0\,..20)$;



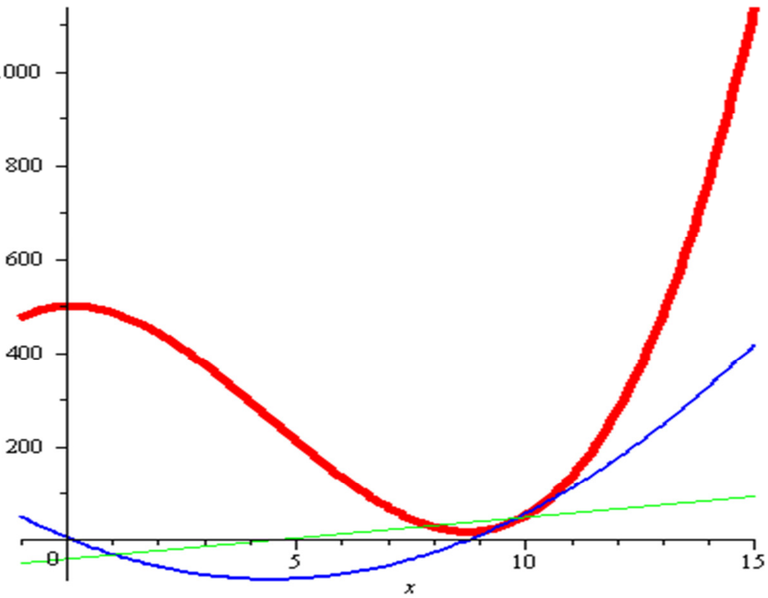
3) Graph with more options (color, line type, and thickness)

> plot(y, x = 0 ..20, color = blue, style = line, thickness = 2);



4) Graphs with more than one equations
Following command will plot the original equation, the first derivative, and the second derivative with different color and different thickness:

> *plot*([*y*, *diff*(*y*, *x*), *diff*(*y*, *x*$2)], *x* = −1 ..15, *color* = [*red*, *blue*, *green*], *thickness* = [5, 2, 1]) ;

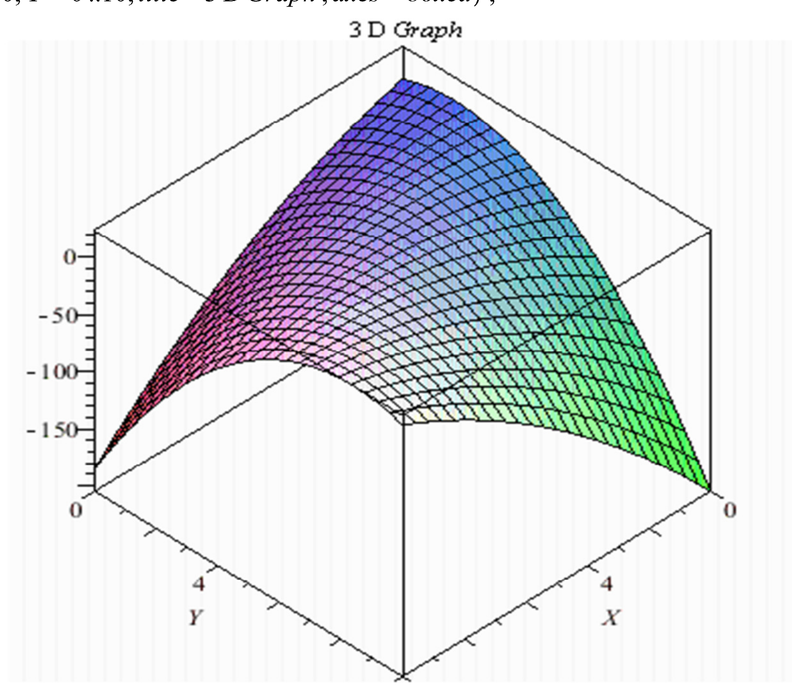


5) 3-Demsional Graph (plot3d)
Plot3d generates the 3 dimensional graphs. The required fields are the equation or predefined equation name, range of two axes. Here is an example of quadratic equation with three variables.

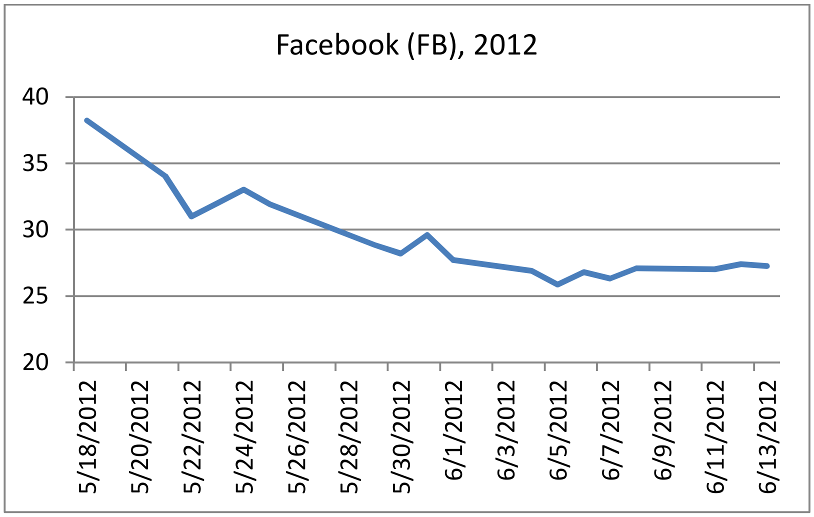
> $Z := -5 - 8*X + 10*Y - X\text{^}2 - 3*Y\text{^}2 + 4*X*Y;$

$$Z := -5 - 8\,X + 10\,Y - X^2 - 3\,Y^2 + 4\,X\,Y$$

> $plot3d(Z, X = 0\,..10,\ Y = 0\,..10, title = \text{'3 D } Graph\text{'}, axes = boxed)\ ;$



Case Study: Facebook IPO (Initial Public Offer) and demand for the Facebook Stock (FB)
Facebook priced its IPO at $38, and NASDAQ listed the stocks and began to trade on May 18. The stock opened at $42, a 10% jump, and spent most of the day trading above $40 thanks to the company's bankders. However the price crashed to under $30 after a week on trade in the market.



1) Suppose the investment bank estimated the following the market demand equation for the Facebook stocks.

$$FB = 4.7 - 0.0618\ P$$
$$\text{Market Cap} = FB * P$$

where FB = number of outstanding share, and P = the price of a share of FB.
As an investment bank, what are the share price for IPO to maximize the value of stocks and the total market capitalization (Market Cap) at the IPO price?

2) We can find many reasons why the Facebook stock price might be overpriced at the time of IPO. Hypothetically, suppose the market demand for the FB was overestimated, and it truly follows the following demand equation:

$$FB = 4.7 - 0.0839\ P$$

Recalculate the share price for IPO to maximize the value of stocks and the total market capitalization at the IPO price?

3) Compare the above two demand equations, the optimal price, and the total market capital at IPO prices.

LAB ASSIGNMENT (MAPLE)

Using Maple, answer the following questions. Before leaving, you need to submit the answers to Dropbox in D2L for lab credit.

I. Solve the following simultaneous equations by using Maple.

1) 20X + 4Y = 280
   10Y – 9X = 110

2) 2X + 7 = 5Y
   3Y + 7 = 4X

II. Make plots and find the first and second derivatives for the following equations. From the first derivate, find the extreme values and find the optimal values

1) $Y = -2(X-5)^2$

2) $Y = \frac{1}{3}X^3 - \frac{3}{2}X^2 + 2X + 1$

III. Make a 3D plot and find the first derivatives for the following equation. From the first derivate, find the Q1 and Q2 to maximize the profit (P). What is the maximum profit?

$$P = -5 - 8Q_1 + 10Q_2 - Q_1^2 - 3Q_2^2 + 4Q_1Q_2$$

IV. Constraint Maximization in Maple

1) Maximize $S = -60 + 140X + 100Y - 10X^2 - 8Y^2 - 6XY$
   Subject to: $20X + 40Y = 200$